CAPITAL BUDGETING USING TERMINAL AMOUNTS

M. F. van Breda

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Comments actively solicited

Current wisdom has it that to evaluate a capital project one should calculate its net present value. If this is positive, one is advised to invest in the project in the absence of restraining qualitative factors. As an alternative, the businessman is sometimes encouraged to calculate internal rates of return. If this exceeds the hurdle rate, the project is acceptable - again all other things being equal. In contrast to the above two methods, the businessman is invariably warned against using the payback method, since this does not take the time value of money into account.

This article suggests a fourth alternative that is rarely stressed in textbooks. Instead of calculating the value of a project as at its start, why not calculate the value of a project as at its close, i.e., its terminal value as opposed to its present value. The advantages of this are several. For one, it highlights the hidden assumptions in present value and internal rate calculations. For another, it makes the comparison of projects much simpler than with any of the methods in current vogue. For a third, it integrates very neatly the time value of money concept with the payback method. It should, however, be stressed that this approach introduces no new theoretical concepts whatsoever.

<u>Accept-reject decisions</u>: We begin with a simple accept-reject capital budgeting decision involving a project with uniform benefits. Assume the investment under consideration extends over four years yielding $10,000 at the end of each year. Assume further that the required initial outlay is $23,616 and that the cost of capital is 10%. Then a simple present value calculation yields the following table.

Table 1

Year	Balance carried down	Receipt on last day of year	Year end balance	Discounted balance
4	0	10,000	10,000	9,091
3	9,091	10,000	19,091	17,356
2	17,356	10,000	27,356	24,869
1	24,869	10,000	34,869	31,699
				Less 23,616
				8,083

The amount \$31,699 is the present value of the project, while \$8,083 is the net present value or NPV. Since the NPV is positive, one is advised to accept the project.

As an alternative to calculating the NPV, one can calculate the internal rate of return or IRR. This is the rate which discounts the benefits to the initial cost of the project. In this case, this is 25% as can be verified from the following table.

Table 2

Year	Balance carried down	Receipt on last day of year	Year end balance	Discounted balance
4	0	10,000	10,000	8,000
3	8,000	10,000	18,000	14,400
2	14,400	10,000	24,400	19,520
1	19,520	10,000	29,520	23,616

If we assume that 10% is the required rate of return of the company, then we should accept this proposal.

It should be obvious that, if the cost of capital is less than the IRR, a positive NPV must result when the cost of capital is used to calculate the NPV. The IRR is merely that rate of interest that drives the NPV to zero. In a sense, it is a special case of the more general NPV calculation. If, for instance, a cost of capital greater than the IRR were used to discount the benefits, a negative NPV would result.

Because of this relationship between the IRR, the cost of capital, and the NPV, the methods give identical results for accept-reject decisions, i.e., one would accept the same projects on either criterion. A positive NPV implies an IRR greater than the cost of capital and vice-versa.

There is, however, no reason why one should compare the costs and benefits of a project as at the <u>outset</u> of that project. One could equally well do the comparison as at the <u>end</u> of the project's life. The following table should make the mechanics involved clear.

Table 3

Year	Balance at start of year	Interest at 10%	Receipt at end of year	Balance at end of year
1	0	0	10,000	10,000
2	10,000	1,000	10,000	21,000
3	21,000	2,100	10,000	33,100
4	33,100	3,310	10,000	46,410
		6,410	40,000	

The amount of $46,410 is the gross terminal amount[1] or GTA of the project. It is the amount that the benefits would have accumulated to if reinvested at the cost of capital of 10%. To arrive at the GTA, one simply calculates the interest that could be earned over the year from the reinvestment of the accumulated benefits and interest to date.

Against the GTA of $46,410 must be set the opportunity cost of the initial investment of $23,616. This is the amount to which the initial investment would have grown if invested at the cost of capital. In this case, this is $34,576 as the following table illustrates.

Table 4

Year	Balance at start of year	Interest at 10%	Balance at end of year
1	23,616	2,362	25,978
2	25,978	2,598	28,576
3	28,576	2,858	31,434
4	31,434	3,142	34,576

Subtracting the opportunity cost of the investment from the GTA yields the net terminal amount or NTA. In this case, this is $46,410 less $34,576 or $11,834. To decide whether to accept or reject the project one uses the following rule.

Rule I: In simple accept-reject decisions if the net terminal amount or NTA is positive, accept the project. Otherwise reject.

To justify this rule, note first that, if the NPV of $8,083 is invested at the cost of capital, it yields the NTA of $11,834.

Table 5

Year	Balance at start of year	Interest at 10%	Balance at end of year
1	8,083	808	8,891
2	8,891	889	9,780
3	9,780	978	10,758
4	10,758	1,076	11,834

A moment's reflection should indicate that this must be so. The NPV is the surplus calculated as at the start of the project. The NTA is the surplus calculated as at the end of the project. Investment of the initial surplus must yield the terminal surplus. Thus, if the NPV is positive, the NTA must be positive. Choices based on positive NTAs will be identical, therefore, to choices based on positive NPVs.

Since the two methods yield identical results, the question may be raised why bother with the terminal calculation. The advantages as we shall see are several. One should be immediately apparent. The GTA of $46,410 is plainly and simply the gross cash the business could expect to have in hand at the end of the project's life. No such interpretation of the present value can be given. Thus, one reason for doing a GTA calculation is its real world meaning.

What should also be apparent is that the IRR is that rate of interest that generates an NTA of zero. This follows from the fact that the NPV invested over the life of the project yields the NTA. If the NPV be zero as a result of using the IRR, then so must the NTA. This is easily confirmed from the following two tables.

Table 6

Year	Balance at start of year	Interest at 25%	Receipt at end of year	Balance at end of year
1	0	0	10,000	10,000
2	10,000	2,500	10,000	22,500
3	22,500	5,625	10,000	38,125
4	38,125	9,531	10,000	57,656

The amount of $57,656 is the accumulated value of the benefits from the project when reinvested at 25%. It is also the amount to which the initial investment of $23,616 would accumulate if invested at 25% compounded annually.

Table 7

Year	Balance at start of year	Interest at 25%	Balance at end of year
1	23,616	5,904	29,520
2	29,520	7,380	36,900
3	36,900	9,225	46,125
4	46,125	11,531	57,656

This last figure of $57,656 is the opportunity cost of the project which when subtracted from the GTA yields the NTA of zero. From this we have a second rule for investment.

Rule II: In simple accept-reject decisions if the rate of interest that yields a net terminal amount of zero exceeds the hurdle rate or the minimum acceptable rate, accept the project. Otherwise reject.

<u>Alternative projects</u>: Consider now the case where we have to choose between two or more projects. Assume by way of example a second project which requires an outlay of $23,616 as before, but now yields a single benefit - an amount of $48,970 at the end of the fourth year.

It is easy to show that this has an NPV of $9,831 at 10%. This derives from a discount factor of 0.683 yielding a present value of $33,447 from which one subtracts the initial outlay of $23,616. Since this is positive, the project is acceptable. Moreover, the NPV of this project is greater than that of the first project which was $8,083. By the usual rule, the second project is to be preferred to the first.

Common sense and the NTA calculation shows the basis for this choice. The net terminal amount of the first project has already been shown to be $11,834. The opportunity cost of the second project is identical to that of the first which was $34,576. The gross terminal amount of the second project is that single, ultimate benefit of $48,970. The net terminal amount of the second project is, therefore, $14,394. Clearly what we have done, therefore, is to choose that project with the highest net terminal amount. We may state this as a rule.

<u>Rule III: Choose the project which yields the greatest net terminal amount.</u>[2]

In cases like the above where the initial investment is identical for both projects, the rule implies that one should choose that project which has generated the most <u>cash</u> by the <u>end</u> of its life. The gross cash yield from the first project was shown in Table 3 to be $46,410 whereas the gross cash yield from the second project is $48,970. All other things being equal, the second project is, therefore, to be preferred.

Consider what this implies. For an individual, viewing a lifetime of benefits and alternative work possibilities say, the rule suggests that he or she wishes to leave the largest bequest possible. For a firm, it implies

that it is wholly indifferent to the cash flow pattern and desires only to maximize its cash holding at the end of the project's life. In our example, it implies that the firm can survive without a steady inflow of cash as in the first project, but can afford to wait for the ultimate large inflow to arrive.

When put this way, many feel that this is not their personal goal, nor is it the goal of their company. In this case, neither a net present value nor a net terminal amount is of any help in choosing a course of action. Both methods assume maximization of end of period wealth as the goal of the investor. The terminal calculation does not offer a new goal. What it does is to make the goal implicit in the NPV calculations much more explicit.

Unfortunately, no general rules can be offered when wealth maximization is rejected as a goal. All one can do is to lay out in some detail the nature of the project and leave the individual to apply his or her own judgement to each specific case. This article argues that terminal calculations provide an ideal framework for doing just that.

<u>Advantages of NTA</u>: It has already been stressed that the one advantage of doing a net terminal amount calculation instead of a net present value calculation is that it makes the implicit goal or criterion much more explicit. The NTA approach makes it quite plain that the assumed aim of the investor is to maximize terminal wealth.

The approach also makes patent the otherwise implicit assumptions with regard to the reinvestment of benefits from the project. It is perfectly clear from Table 3 that we are assuming that benefits can and will be reinvested at the cost of capital. This fact is not at all obvious from an examination of the equivalent present value calculation in Table 1. Again, the terminal value method introduces no new theory, but does highlight the

assumptions in given theory.

This last point enables one to understand immediately why the IRR criterion and the NPV criteria sometimes clash when evaluating two projects. Consider again the two projects above. The IRR on the first project has already been shown to be 25%. One can easily show that the IRR on the second project is 20%. The four year discount factor appropriate to a rate of 20% is 0.482 which multiplied by $48,970 gives $23,616. Subtracting the initial outlay yields an NPV of zero.

One might be inclined on this basis to favor the first project since its IRR is five points higher. Recall, however, that the NPV calculation indicates that the second project is to be preferred. The reason for this apparent anomaly lies in the assumptions made about the reinvestment of benefits - implicit in the NPV calculation, explicit in the NTA calculation.

The IRR is that rate which results in an NTA of zero. In the case of the first project, this is 25% as was demonstrated in Tables 6 and 7 above. However, these tables assume that benefits will be reinvested at the rate of 25%. By contrast, if we repeat the IRR calculation as in those tables, but for the second project, one assumes that benefits are reinvested at 20% only. And, more importantly still, the original NPV calculations in Table 1 assumed that the benefits could and would be reinvested at the much lower cost of capital of 10%. In other words, the cause of the anomaly lies in the hidden switch of assumptions - a switch which can be made but which is not hidden when doing NTA calculations.

To see the numerical effect, note, from Table 6 above, that the first project will accumulate to $57,656 at 25%. This greatly exceeds the GTA of the second project which is $48,970. At this rate, therefore, the first project is to be preferred. However, at a rate of 10%, the GTA of the first project is only $46,410, as is evident is Table 3, and the second

project is preferable.

As a third benefit, the terminal approach enables one to lay the project information out in a thoroughly satisfactory manner when the single maximization goal is deemed inappropriate. For example, the first project could be laid out for management's evaluation in this form.

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ar	Cash balance at start of year	Interest at 10%	Receipt at end of year	Cash inflow	Gross balance at end of year	Net cash balance at end of yea
1	0	0	10,000	10,000	10,000	-13,616
2	10,000	1,000	10,000	11,000	21,000	- 2,616
3	21,000	2,100	10,000	12,100	33,100	9,484
4	33,100	3,310	10,000	13,310	46,410	22,794

The last column in this table indicates the cash balance at the end of each year, after making allowance for the initial investment of $23,616. It shows the cash in hand from the benefits generated by the project and from the interest earned by reinvesting those benefits. Not only is this provided for the end of the project, but for each year of its life.

Not only does the table provide a full picture of the cash flows associated with the project, but it also indicates at what point the receipts alone and more particularly, the receipts plus interest will exceed the initial outlay. In other words, the payback period is readily apparent from the last column of this table.

By extending the table a few more columns or by providing an additional table, as is done here, one can compare the benefits received from the project with the opportunity cost of the project, i.e.,

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ar	Gross balance at end of year	Cash outflow	Net cash balance at end of year	Opportunity cost	Net terminal va
1	10,000	23,616	-13,616	2,362	-15,978
2	21,000	-	- 2,616	4,960	- 7,576
3	33,100	-	- 9,484	7,818	1,666
4	46,410	-	22,794	10,960	11,834

The opportunity cost in this table is the interest lost by investing in the project rather than putting the amount of $23,616 into 10% bonds say. The amounts are cumulative and are drawn from Table 4 above.

It is worth noting how including the opportunity cost delays the break-even point in the project. The last column indicates the net terminal value at the end of each year. This is an added bonus from setting the calculations out in this way. One can see not only when cash will break even, but also when benefits versus costs will break even. In the light of earlier comments, it might be noted here that on a cost-benefit basis at the IRR of 25%, the project does not break even until the end of the project's life. This is, of course, due to the fact that at a rate of interest of 25% opportunity costs are much higher than at a rate of interest of only 10%.

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Balance at start	Interest at 25%	Receipt at end of year	Opportunity cost at start of yr.	Interest at 25%	Net ter min val
0	0	10,000	-23,616	- 5,904	-19,5
10,000	2,500	10,000	-29,520	- 7,380	-14,4
22,500	5,625	10,000	-36,900	- 9,225	- 8,0
38,125	9,531	10,000	-46,125	-11,531	

Finally, one other advantage of this approach is that it enables one to easily and simply make more general assumptions about the project. One can, for example, assume that the reinvestment rate will decline as in the following table.

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Balance at start of year	Interest rate	Interest	Receipt at end of year	Balance at end of year
0	-	-	10,000	10,000
10,000	25%	2,500	10,000	22,500
22,500	20%	4,500	10,000	37,000
37,000	10%	3,700	10,000	50,700

Of course, varying interest rates can be used with NPV calculations. The difference is that here one can see exactly and explicitly what is being assumed.

To summarize then - the whole theory of the time value of money assumes that one wishes to maximize net terminal wealth. All that net present value does is to discount this goal back to the here and now and label it net present value. This does not, of course, alter the goal. This aim of maximizing end of period wealth may or may not be appropriate. That depends on circumstances. What this article suggests is that, by setting the calculations out in investment terms rather than in discounting terms, the maximization goal is made explicit. Moreover, it enables the reader to see the annual gross cash flow picture, the cash break even point, and the cost-benefit break even point. All these are vital pieces of information which should not be excluded from any project evaluation.

FOOTNOTES

1. The use of the word "amount" is deliberate. Value has connotations of a measure of utility, which are not implied here at all. It would also be preferable, for the same reason, to speak of present amounts, but the term present value is so rooted in the language now that a change in terminology is unlikely. One hopes that the less familiar terminal value will be replaced by the more correct terminal amount.

2. An early statement of this rule may be found in Solomon (1956) who wrote:

   "The valid comparison is not simply between two projects but between two alternative courses of action. The ultimate criterion is the total wealth that the investor can expect from each alternative by the terminal date of the longer-lived project."

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